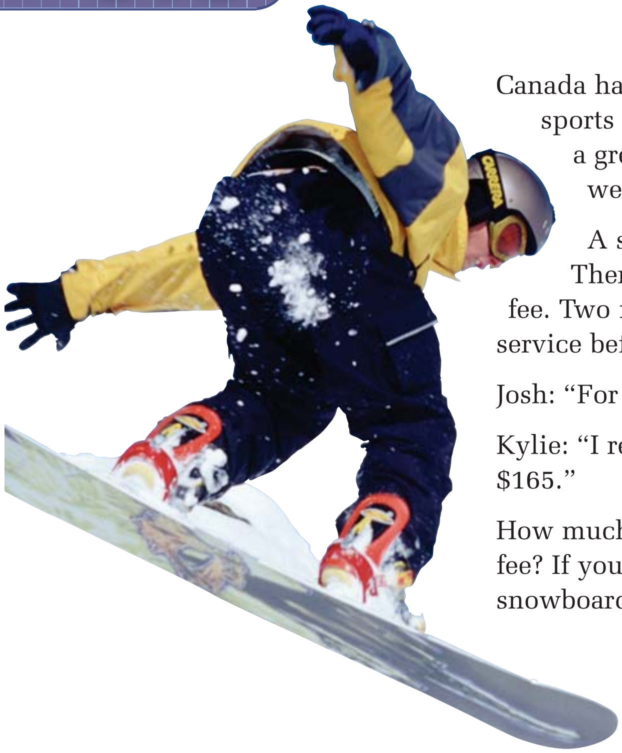
6.6 Find an Equation for a Line Given Two Points



Canada has some very long winters. Enjoying winter sports such as snowboarding, hockey, and skiing is a great way to make the most of the cold weather!

A ski resort rents snowboards by the day. There is a flat insurance cost, plus a daily rental fee. Two friends, Josh and Kylie, have used the rental service before. They compare costs.

Josh: "For 3 days, the snowboard rental cost me \$85."

Kylie: "I rented a board for a full week, and it cost me \$165."

How much are the insurance cost and the daily rental fee? If you have \$120, can you afford to go snowboarding for 5 days?

Investigate

Tools

- grid paper
- ruler

Making Connections

You studied direct and partial variations in Chapter 5: Modelling With Graphs. Is this a direct or a partial variation? How do you know?

How can you construct a linear model if you know two points of information?

See the information above about renting a snowboard.

1. a) On grid paper, draw and label two sets of axes with
- cost, C, in dollars, on the vertical axis
- time, d, in days, on the horizontal axis

b) Add appropriate scales to your axes to fit the data in the introduction.

2. a) Plot the points (3, 85) and (7, 165) and explain what they mean.

b) Draw a line through these points. Extend the line so that it crosses the vertical axis.

3. a) Find the slope of this line and explain what it means.

b) Find the C-intercept and explain what it means in terms of renting a snowboard.

4. Write the equation of the line in the form $C = md + b$, where m is the slope and b is the C-intercept.

5. **a)** Use the graph to find the cost of renting a snowboard for 5 days.
   b) Use your equation from step 4 to find the cost of renting a snowboard for 5 days.
   c) Are these answers the same? Explain.
6. **Reflect** Is it possible to find an equation for this line without graphing it? Explain.

Linear models can be useful for making predictions in many situations involving direct and partial variation. If you know two points of information, you can find an equation for the line.

Step 1. Find the slope. Substitute the two known points into the slope formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Step 2. Find the y-intercept. Substitute the slope and one of the two points into $y = mx + b$. Solve for b.

Step 3. Write the equation. Substitute the slope and y-intercept into $y = mx + b$.

Example Find an Equation for a Line, Given Two Points

a) A line passes through (1, 2) and (5, 10). Find an equation for the line.

b) A line passes through (−3, −2) and (6, −8). Find an equation for the line.

Solution

a) *Step 1*. Find the slope.

Substitute (1, 2) and (5, 10) into the slope formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

$$m = \frac{10 - 2}{5 - 1}$$
$$= \frac{8}{4} \text{ or } 2$$

The slope is 2.

Step 2. Find the y-intercept.

Substitute $m = 2$ and one of the points, say (1, 2), into $y = mx + b$.

$2 = 2(1) + b$
$2 = 2 + b$
$0 = b$

The y-intercept is 0.

Step 3. Write the equation.

Substitute $m = 2$ and $b = 0$ into $y = mx + b$.

$$\begin{aligned} y &= (2)x + (0) \\ &= 2x + 0 \\ &= 2x \end{aligned}$$

The equation of the line passing through (1, 2) and (5, 10) is $y = 2x$.

b) *Step 1.* Find the slope.

Substitute (−3, −2) and (6, −8) into the slope formula.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-8 - (-2)}{6 - (-3)} \\ &= \frac{-8 + 2}{6 + 3} \\ &= \frac{-6}{9} \\ &= -\frac{2}{3} \end{aligned}$$

Simplify integer calculations.

The slope is $-\frac{2}{3}$.

Step 2. Find the y-intercept.

Substitute $m = -\frac{2}{3}$ and one of the points, say (6, −8), into $y = mx + b$.

$$-8 = \left(-\frac{2}{\cancel{3}_1}\right)(\cancel{6}^{2}) + b$$

$$\begin{aligned} -8 &= -4 + b \\ -8 + 4 &= b \\ -4 &= b \end{aligned}$$

The y-intercept is −4.

Step 3. Write the equation.

$y = mx + b$

$$y = -\frac{2}{3}x + (-4)$$

Substitute $m = -\frac{2}{3}$ and $b = -4$.

The equation of the line is $y = -\frac{2}{3}x - 4$.

Key Concepts

- You can find an equation for a line if you know two points on the line. To find the equation,
  - find the slope by substituting the two points into the slope formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
  - find the y-intercept by substituting the slope and one of the points into $y = mx + b$, and then solve for b
  - write the equation by substituting m and b into $y = mx + b$

Communicate Your Understanding

C1 Explain how you can find an equation for a line if you are given

a) the slope and the y-intercept

b) the slope and a point on the line

c) two points on the line

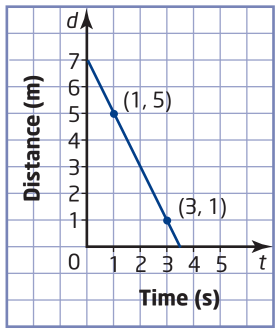
C2 Create an example of each type in question C1 to illustrate your explanation.

C3 Suppose that you know two points on a line: (1, 2) and (−3, −2).

a) Once you have found the slope, investigate whether it matters which point you substitute into $y = mx + b$.

b) Which point would you prefer to use, and why?

C4 The graph illustrates a walker's movement in front of a motion sensor. Answer true or false to the following statements, and explain your answers.



a) The walker started at a distance of 1 m from the sensor.

b) After 3 s, the walker was 1 m from the sensor.

c) The walker's speed was 2 m/s toward the sensor.

C5 The method you follow to write an equation for a line differs depending on the information you are given. Summarize the steps you would use to write the equation of a line given the following information:

a) two points on the line

b) one point on the line and its y-intercept

c) the x- and y-intercepts

Practise

For help with questions 1 to 4, see Example 1.

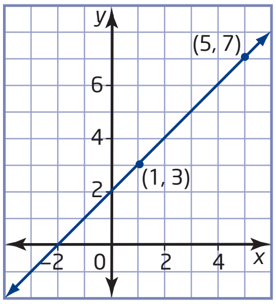
1. Find an equation for the line passing through each pair of points.

a) P(2, 3) and Q(5, 6) **b)** A(4, −1) and B(0, 5)

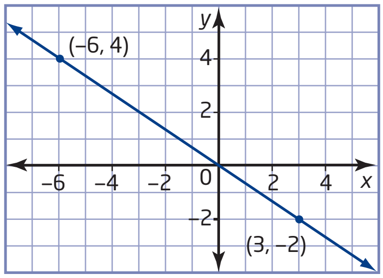
c) U(−3, 4) and V(−2, −6) **d)** L$\left(\frac{1}{2}, 0\right)$ and M$\left(\frac{7}{2}, -5\right)$

2. Find an equation for each line.

a)



b)



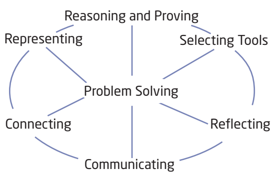
3. a) Find an equation for the line with an *x*-intercept of 4 and a *y*-intercept of −2.

b) Find an equation for the line whose *x*- and *y*-intercepts are both −5.

4. Find the equation of a line passing through each pair of points.

a) M(0, 3) and N(5, 3) **b)** K(−2, 6) and L(−2, −4)

Connect and Apply



5. A bowling alley has a fixed base cost and charges a variable per game rate. It costs \$20.50 for five games and \$28.50 for nine games.

a) What is the variable cost?

b) Find an equation for the line relating cost, *C*, in dollars, and number of games, *g*, in the form $C = mg + b$.

c) Graph this linear relation.

d) What is the *C*-intercept? What does it mean?

e) Use the graph to find the cost of 20 games.

f) Use the equation to find the cost of 20 games.

g) Describe one advantage and one disadvantage of using
- the graph
- the equation

6. Fiona is walking at a constant speed in front of a motion sensor. After 2 s, she is 1.5 m from the sensor. 2 s later, she is 4.5 m from the sensor.

a) Is Fiona moving toward or away from the sensor? Explain how you know.

b) How fast is Fiona walking?

c) Find the equation that describes Fiona's motion in the form $d = mt + b$.

d) What is the *d*-intercept? What does it mean?

7. Workers at a laboratory get the same raise each year. Colette, who has been working at the lab for 5 years, earns \$17.25/h. Lee, who has been working at the lab for 1 year, earns \$14.25/h. The equation relating wage and number of years worked is of the form $w = mn + b$, where w is the hourly wage and n is the number of years worked.

a) (5, 17.25) and (1, 14.25) are two points on the line. Explain why.

b) Find the slope and the w-intercept of this line, and explain what they mean.

c) Write the equation of the line.

d) Maria has been working at the lab for 7 years. Determine her hourly wage.

e) What wage does this linear model predict for a worker who has been with the lab for 25 years? Does this seem reasonable? Explain. How might the store modify the raise policy?

8. Anil's family is driving home to Toronto. Anil hopes that they will make it back in time to see the hockey game on television. While travelling at a fairly constant speed, he observes two signs along the trip.

At 4:30 P.M.:

At 7:00 P.M.:

a) How fast is Anil's family travelling?

b) Find a linear equation that relates distance from home, in kilometres, to time travelled, in hours.

c) The game starts at 7:45 P.M. Will they make it back to Toronto in time? If yes, how much spare time will Anil have to make it to the TV? If not, how late will he be? What assumptions must you make?

Extend

9. Two students are walking at constant speeds in front of two motion sensors.

- Lucas starts at a distance of 6 m and, after 10 s, he is 1 m away from his sensor.
- Myrna starts at a distance of 2 m and, after 8 s, she is 6 m from her sensor.

a) Find a distance-time equation for each walker.

b) At what time were they at the same distance from their sensors?

c) At what distance did this occur?

d) Explain how you solved parts b) and c).

10. Refer to question 9.

a) Graph both linear relations on the same grid.

b) Identify the point where the two lines cross. This is called the point of intersection. What are the coordinates of this point?

c) Compare this point to your answers to question 9, parts b) and c). Explain what you notice.